Global Temperature Change and Sea-Level Rise

M. M. Aral, J. Guan and B. Chang
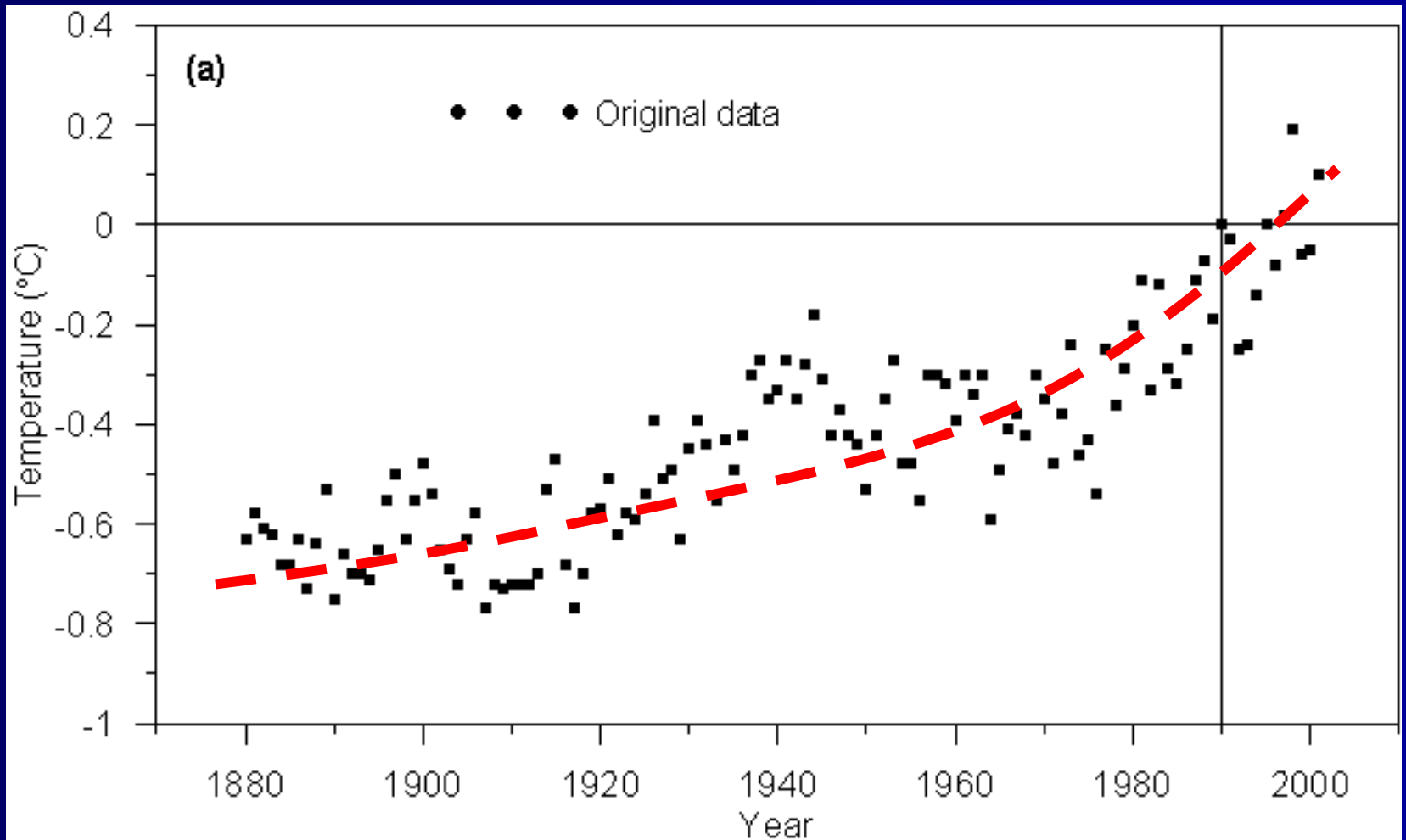
Multimedia Environmental Simulations Laboratory
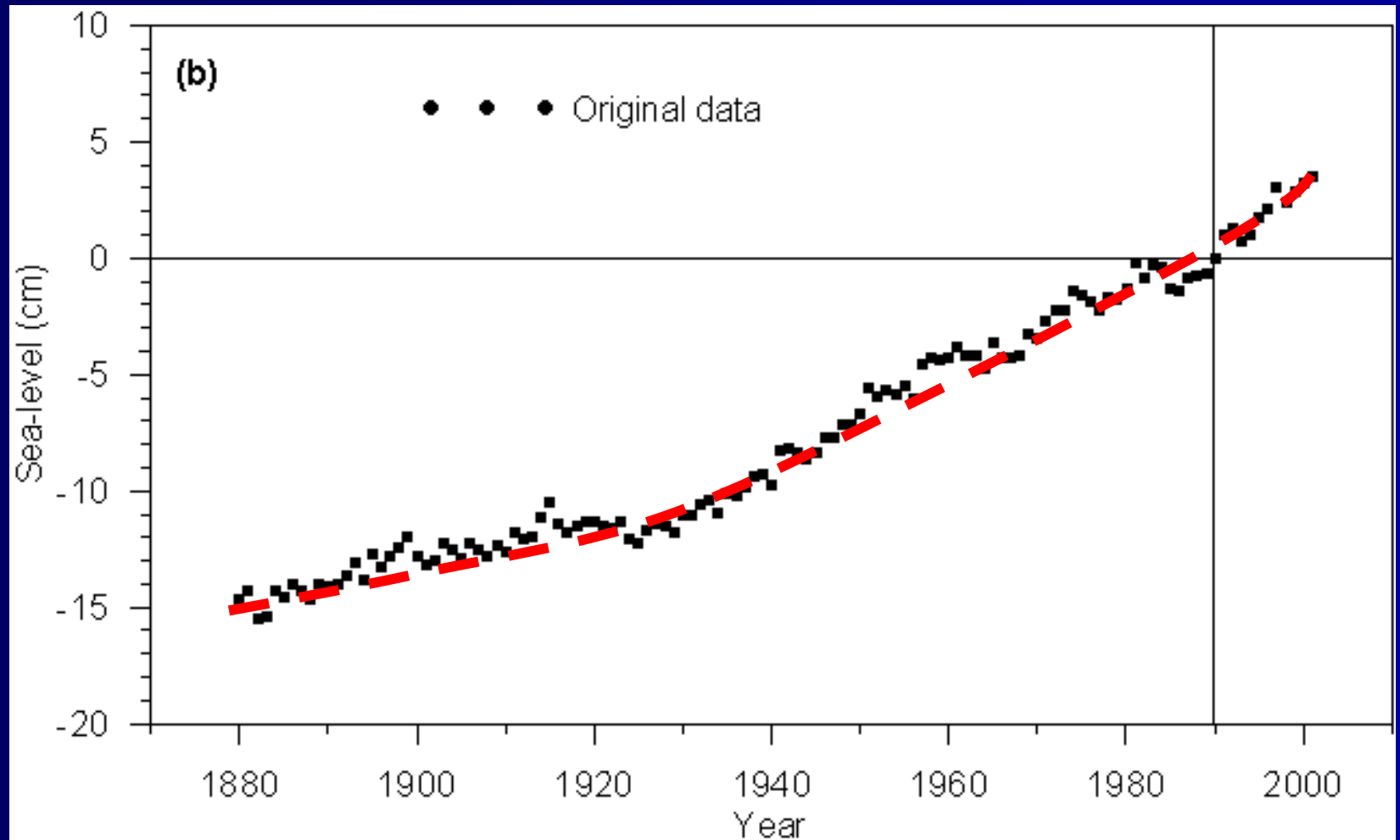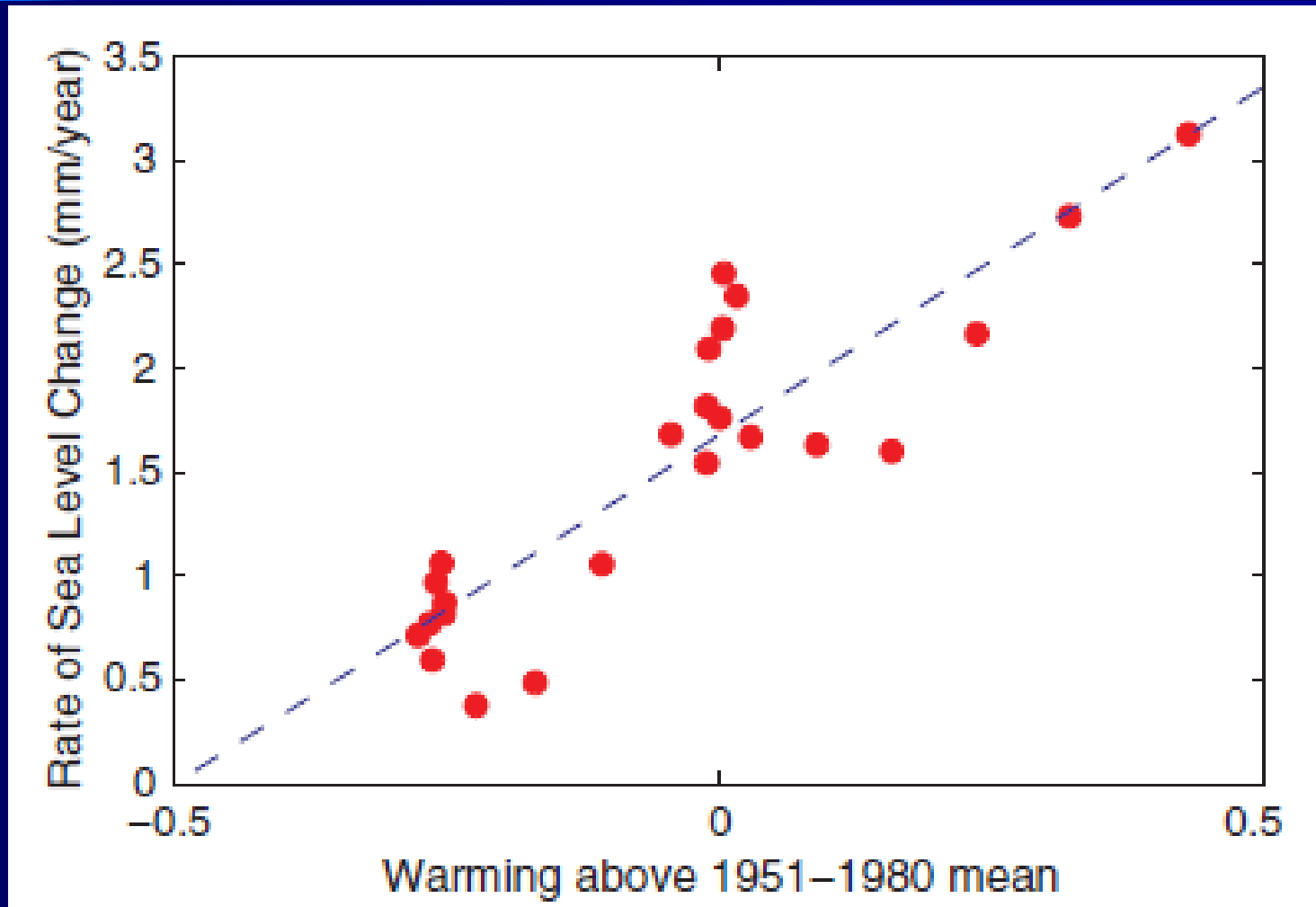School of Civil and Environmental Engineering
Georgia Institute of Technology

Global Temperature Change:

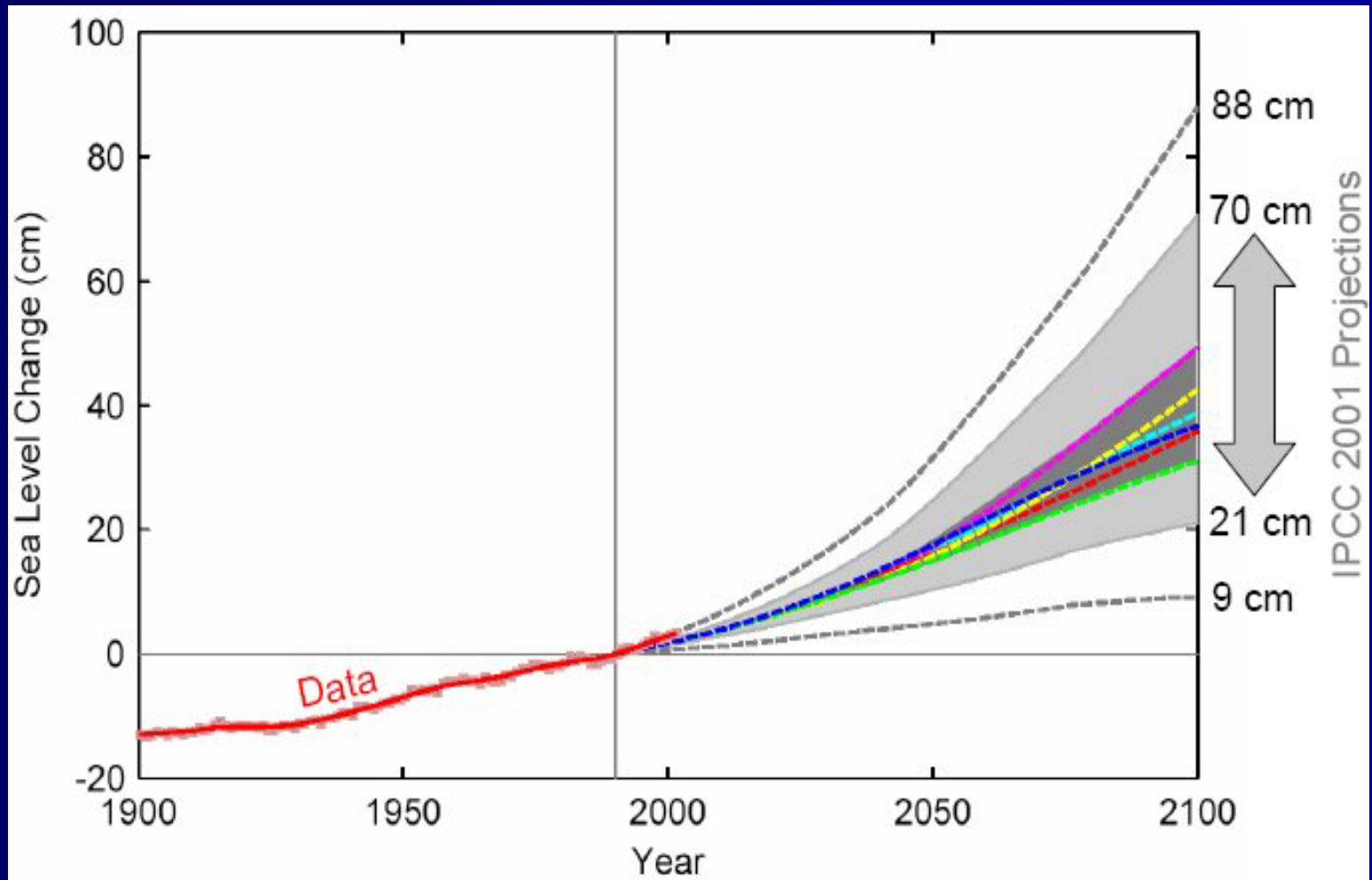# Global Sea-Level Rise:

Correlation:

What does IPCC say?

The Summary for Policy Makers (SPM) released recently provides the following table of sea level rise projections:

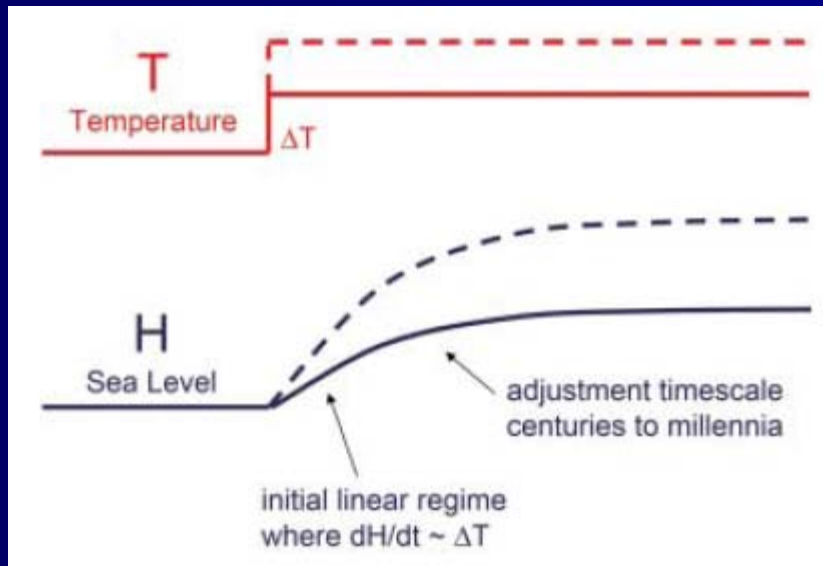| Case	Sea Level Rise (m at 2090-2099 relative to 1980-1999) Model-based range excluding future rapid dynamical changes in ice flow
B1 scenario	0.18 – 0.38
A1T scenario	0.20 – 0.45
B2 scenario	0.20 – 0.43
A1B scenario	0.21 – 0.48
A2 scenario	0.23 – 0.51
A1FI scenario	0.26 – 0.59

What does IPCC say?

Semi-empirical models:

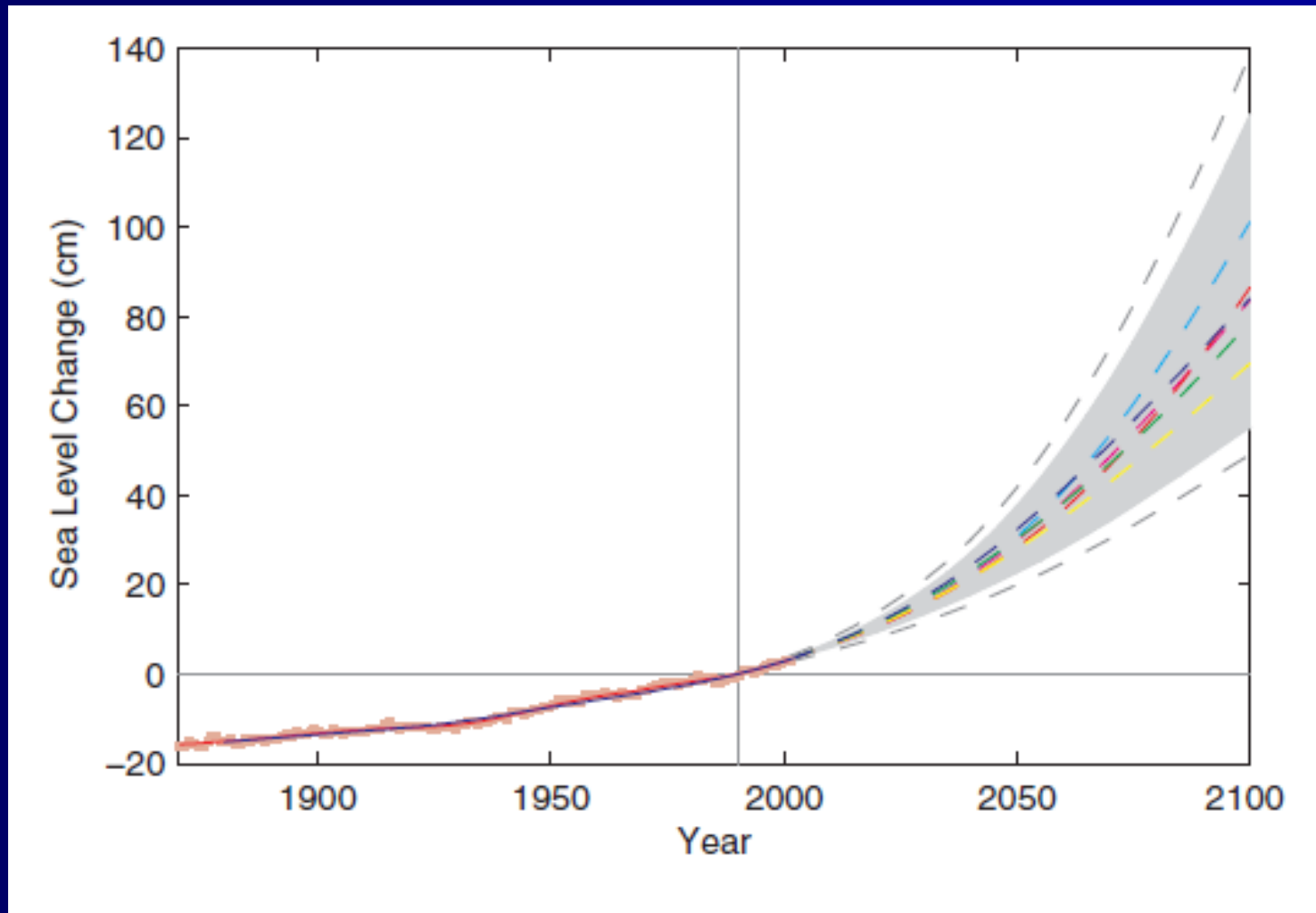**Rahmstorf's Study (Science, Vol. 315 pp.19, 2007)**

$$\frac{\partial H}{\partial t} = a\left(T - T_o\right)$$

$$H(t) = a\int_{t_o}^{t}\left(T(\tau) - T_o\right)d\tau$$

Semi-empirical models:

Rahmstorf's Study   (Science, Vol. 315 pp.19, 2007)

Semi-empirical models:

Rahmstorf's Study   (Science, Vol. 315 pp.19, 2007)
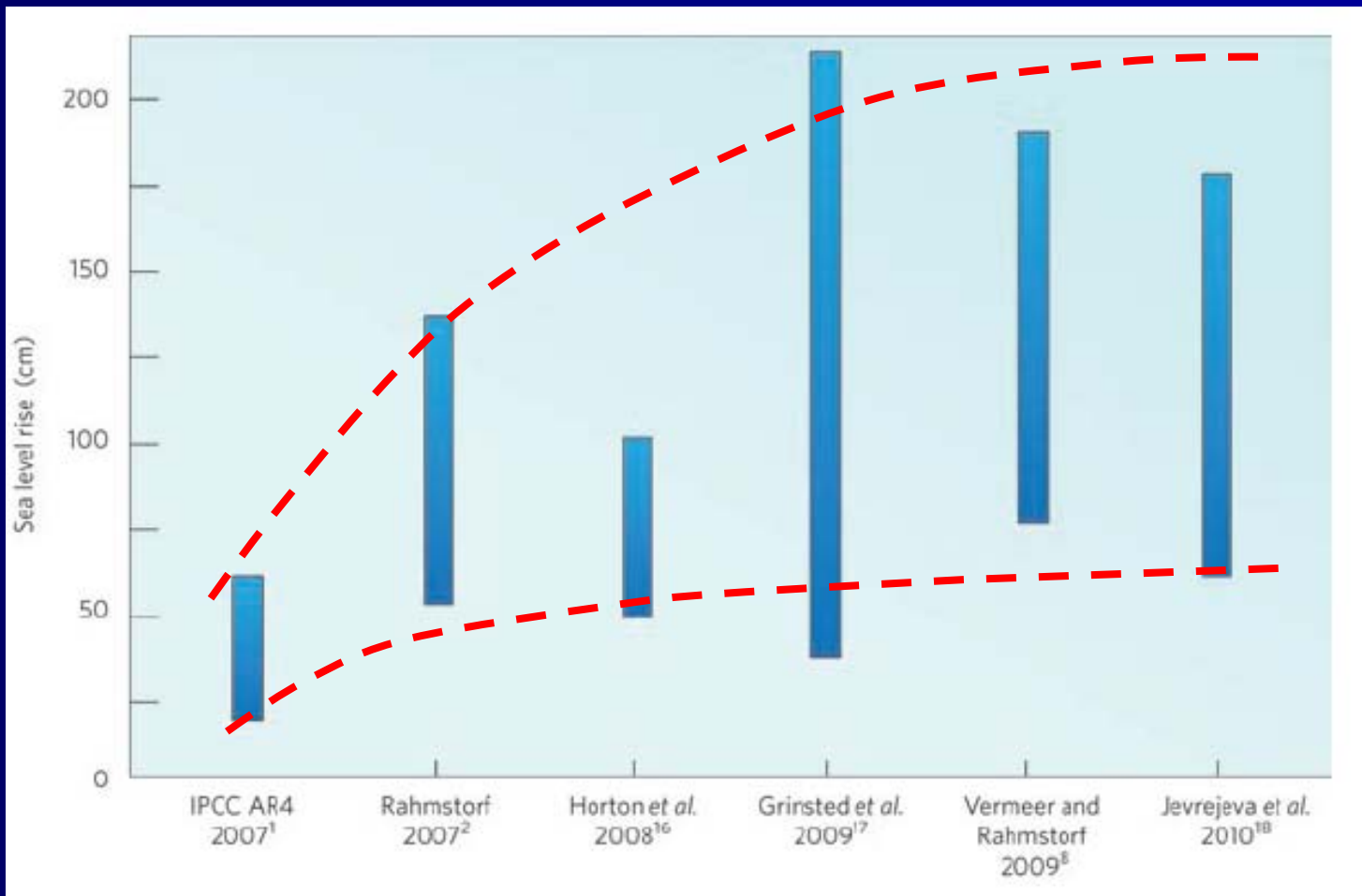
Sea-Level Rise Interval Predicted:

$\{0.5 - 1.4m\}$ above 1990 level

IPCC Interval Predicted:

$\{0.09 - 0.88m\}$ above 1990 level

IPCC and other Models:

Everything should be made as simple as possible, but not simpler.

A. Einstein

A Dynamic Systems Model

Expected Relationship:

- Earlier studies showed that the relationship between T & H is linear.

Hypothesis:

- Temperature: $T = f_1\left(T, H, U, c_1\right)$

- Sea-Level: $H = f_2\left(T, H, U, c_2\right)$

Proposed Model:

$$\frac{dT(t)}{dt} = a_{11}T(t) + a_{12}H(t) + a_{13}U_i(t) + c_1$$

$$\frac{dH(t)}{dt} = a_{21}T(t) + a_{22}H(t) + a_{23}U_i(t) + c_2$$

Proposed Model: (Simplified)

$$\frac{dT(t)}{dt} = a_{11}T(t) + a_{12}H(t) + c_1$$

$$\frac{dH(t)}{dt} = a_{21}T(t) + a_{22}H(t) + c_2$$

Proposed Model:

$$\mathbf{X}(t) = \big(T(t),\, H(t)\big)^{\tau} = \big(x_1(t),\, x_2(t)\big)^{\tau}$$

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad \mathbf{C} = \begin{Bmatrix} c_1 \\ c_2 \end{Bmatrix}$$

$$\frac{d\mathbf{X}}{dt} = \mathbf{A}\mathbf{X}(t) + \mathbf{C}$$

Proposed Discrete Model:

$$\mathbf{X}(k+1) = \mathbf{\Phi}\,\mathbf{X}(k) + \mathbf{\Omega}\,(k) +$$

$$\mathbf{\Phi} = \left(\mathbf{I} + \mathbf{A}\Delta t\right); \quad \mathbf{\Omega} = \mathbf{C}\Delta t$$

LSM Model to determine a_{ij} & c_i :

$$F^* = \underset{\varphi_i}{\text{minimize}} \left\{ \left(\mathbf{Y}_i \mathbf{\Lambda} \mathbf{\varphi}_i \right)^T \mathbf{Y} \left(_i \mathbf{\Lambda} \mathbf{\varphi}_i \right) \right\}$$

Confidence Interval:

$$\hat{T}_{CI}(k) = \hat{T}(k) \pm t_{\alpha/2, n-4} \sqrt{\hat{\sigma}_T^2 \left(1 + \hat{Z}(k)^\tau \left(\mathbf{\Lambda}^\tau \mathbf{\Lambda} \right)^{-1} \hat{Z}(k) \right)}$$

$$\hat{H}_{CI}(k) = \hat{H}(k) \pm t_{\alpha/2, n-4} \sqrt{\hat{\sigma}_H^2 \left(1 + \hat{Z}(k)^\tau \left(\mathbf{\Lambda}^\tau \mathbf{\Lambda} \right)^{-1} \hat{Z}(k) \right)}$$
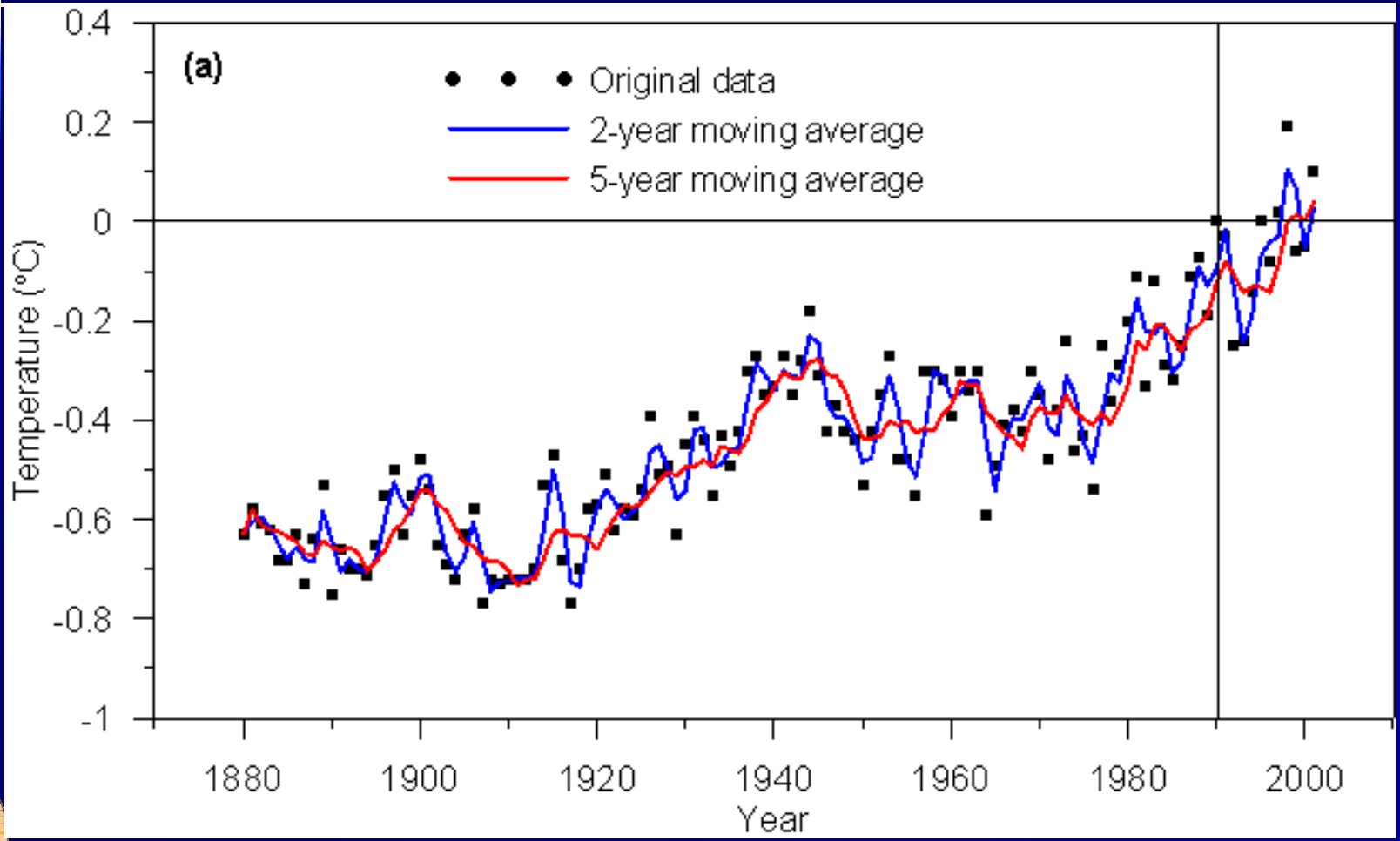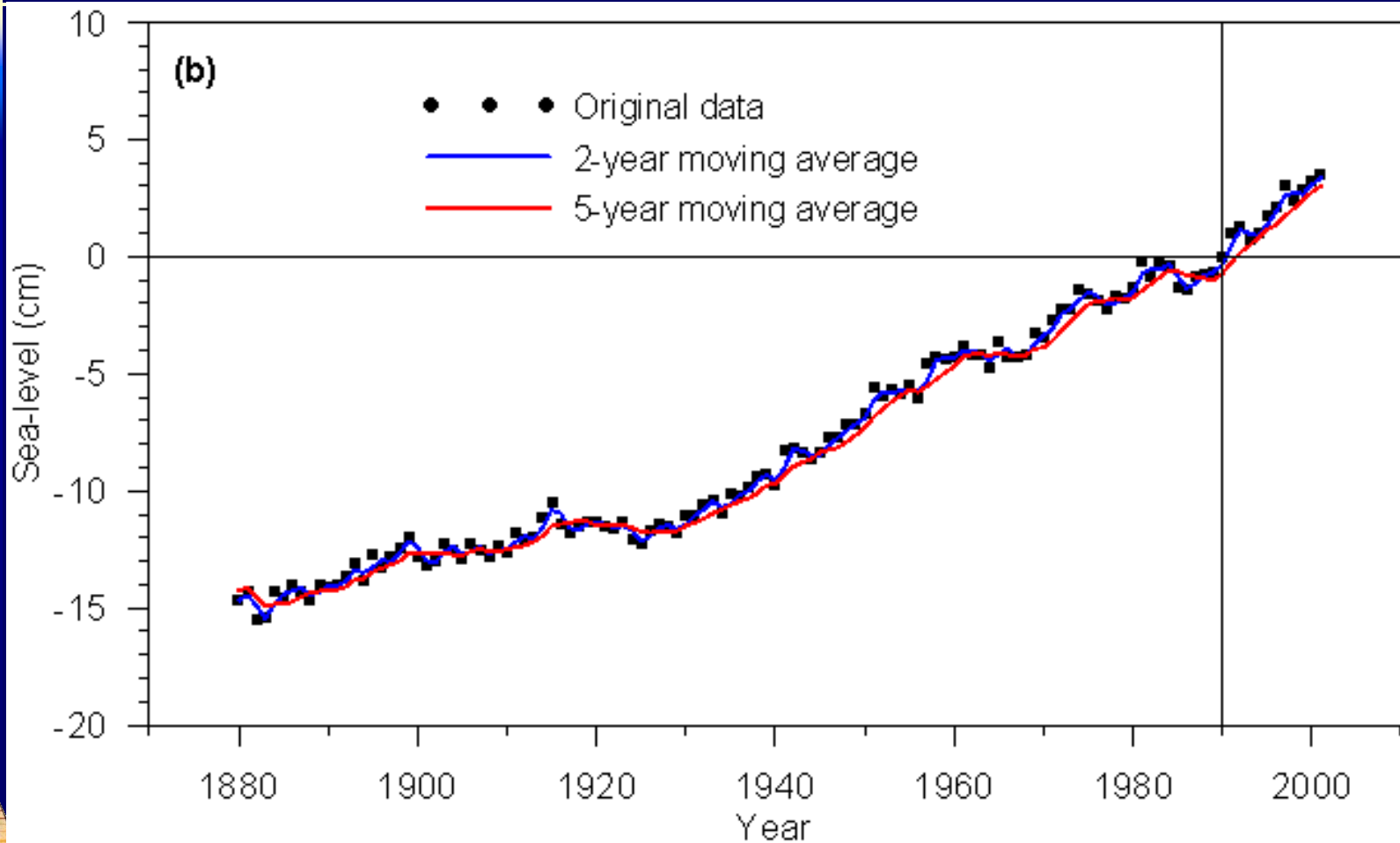
Application:

- 2-year moving average outcome is used for both state variables.
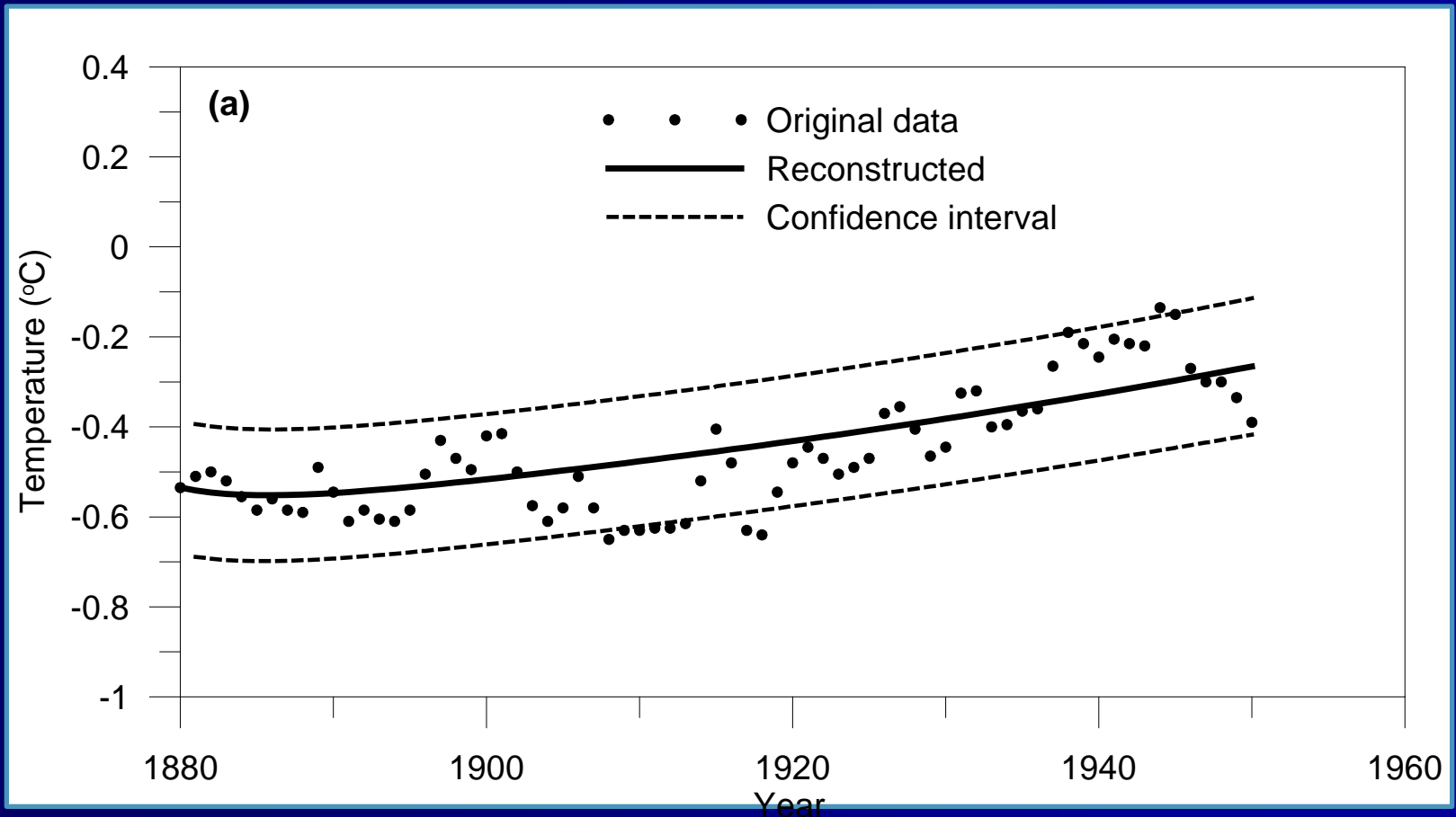
Temperature Data:

Sea-Level Data:

Matrix Coefficients:

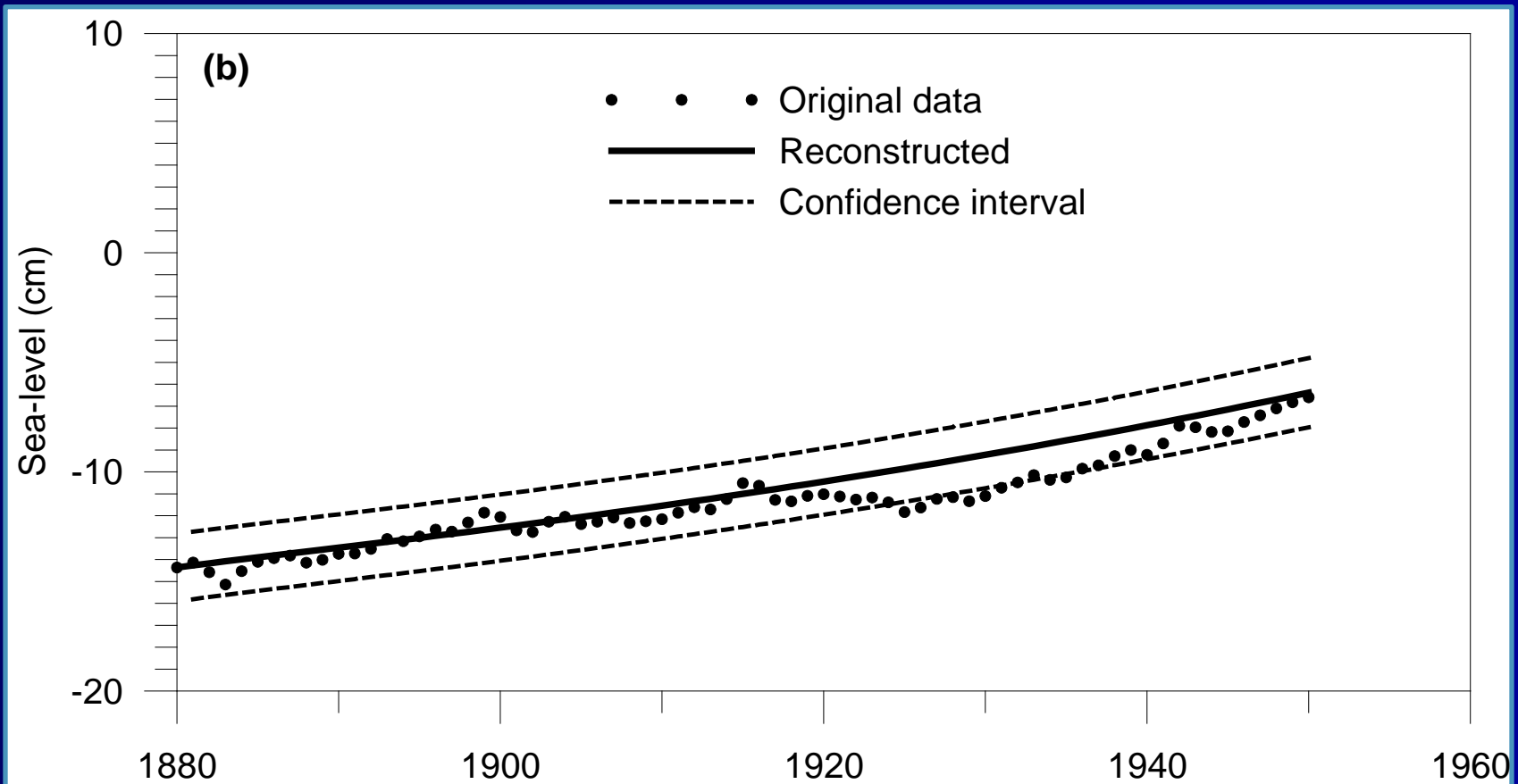| Data used	Discrete system (Φ, Ω)		Continuous system (A, C)	
1880 - 1950	$\begin{bmatrix} 0.8337 & 0.0072 \\ 0.3441 & 0.9960 \end{bmatrix}$	$\begin{Bmatrix} 0.0083 \\ 0.2234 \end{Bmatrix}$	$\begin{bmatrix} -0.1663 & 0.0072 \\ 0.3441 & -0.0040 \end{bmatrix}$	$\begin{Bmatrix} 0.0083 \\ 0.2234 \end{Bmatrix}$
1880 - 2001	$\begin{bmatrix} 0.8074 & 0.0068 \\ 0.4115 & 0.9956 \end{bmatrix}$	$\begin{Bmatrix} -0.0110 \\ 0.2585 \end{Bmatrix}$	$\begin{bmatrix} -0.1926 & 0.0068 \\ 0.4115 & -0.0045 \end{bmatrix}$	$\begin{Bmatrix} -0.0110 \\ 0.2585 \end{Bmatrix}$

Temperature (1880-1950): (90%) Conf. Int.

(a)

- • • • Original data
- —— Reconstructed
- ------ Confidence interval

Temperature (°C) vs Year

$R^2 = 0.6$

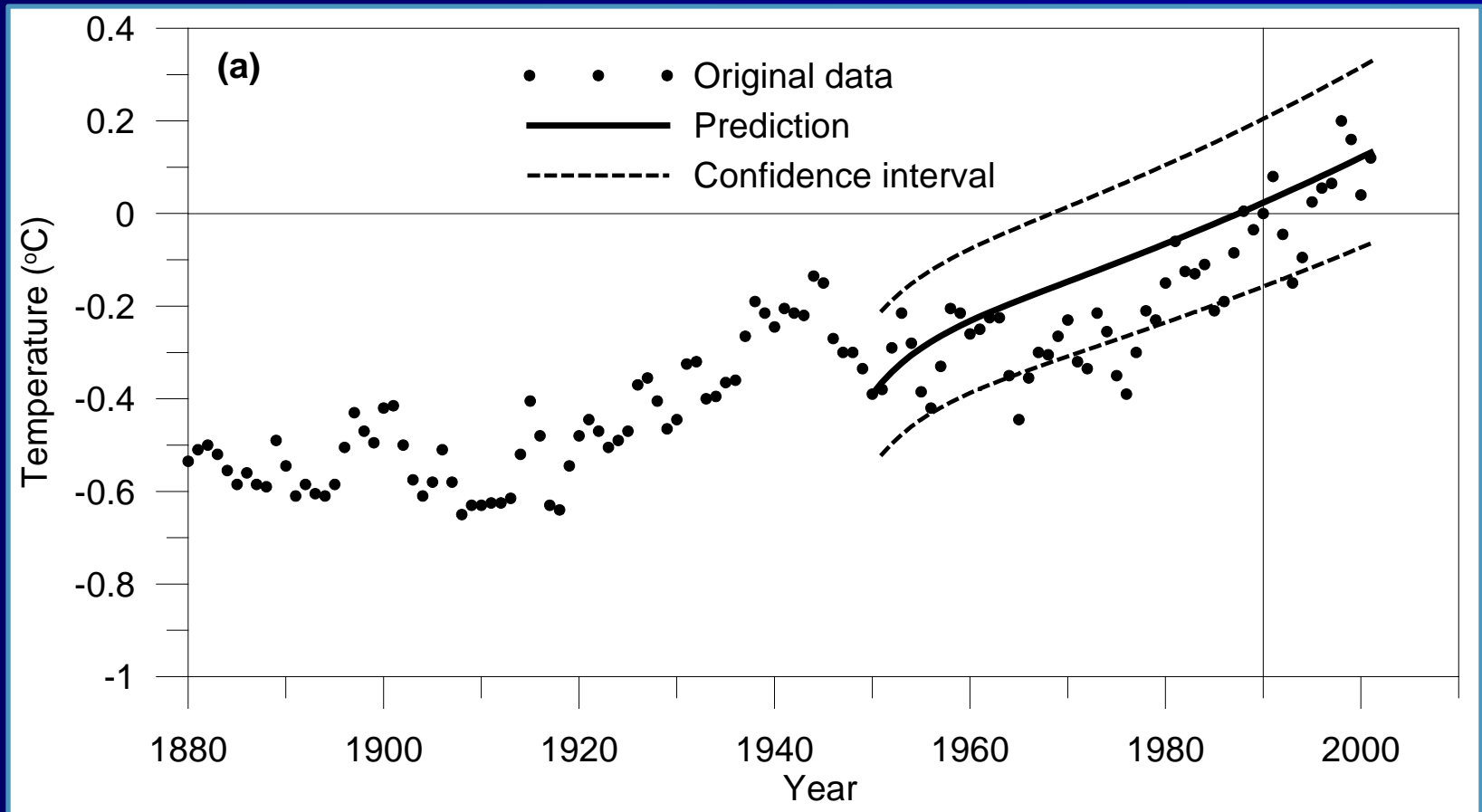# Sea-Level (1880-1950): (90%) Conf. Int.
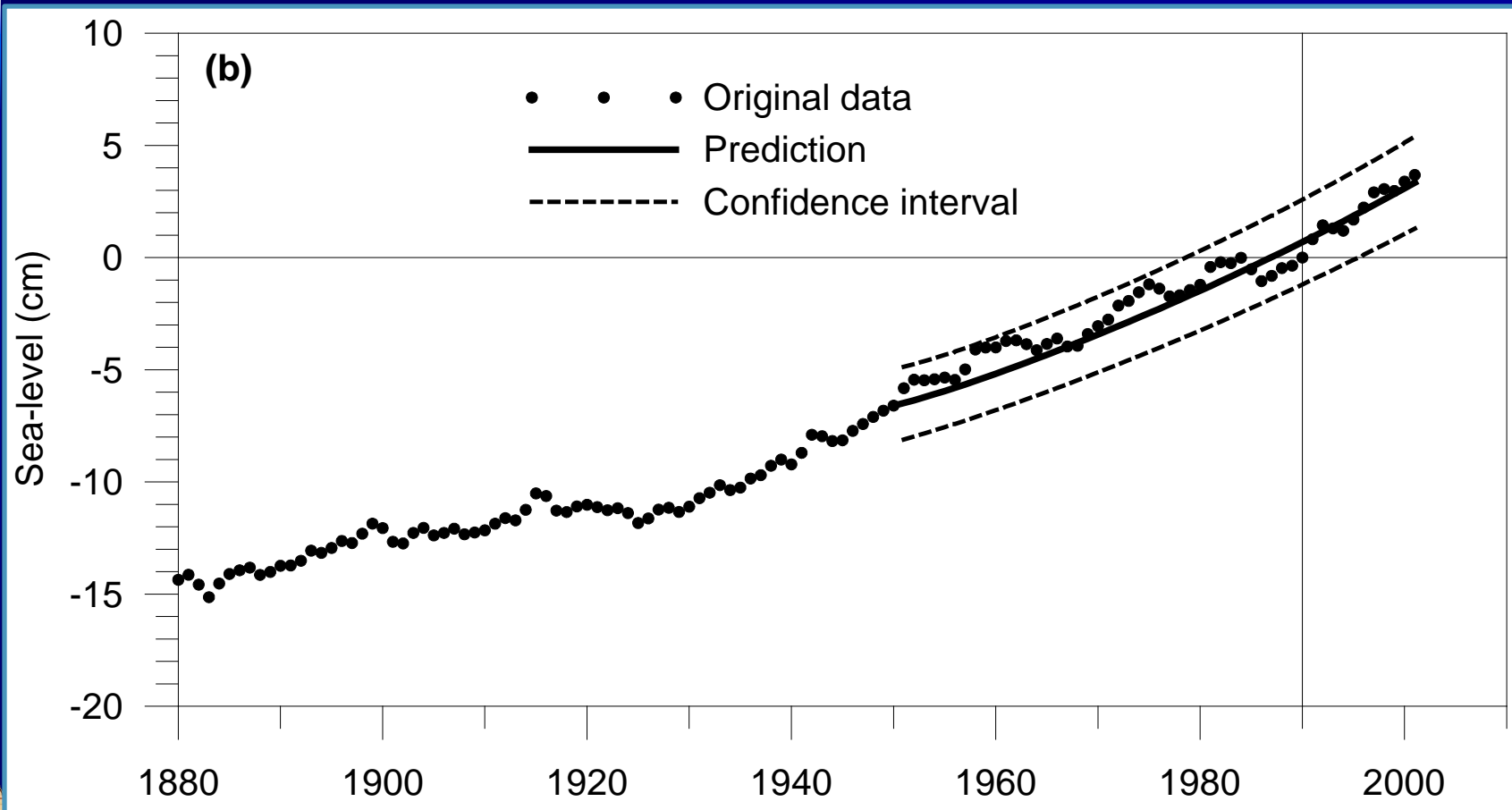

R² = 0.8

Temperature (1950-2001): (90%) Conf. Int.
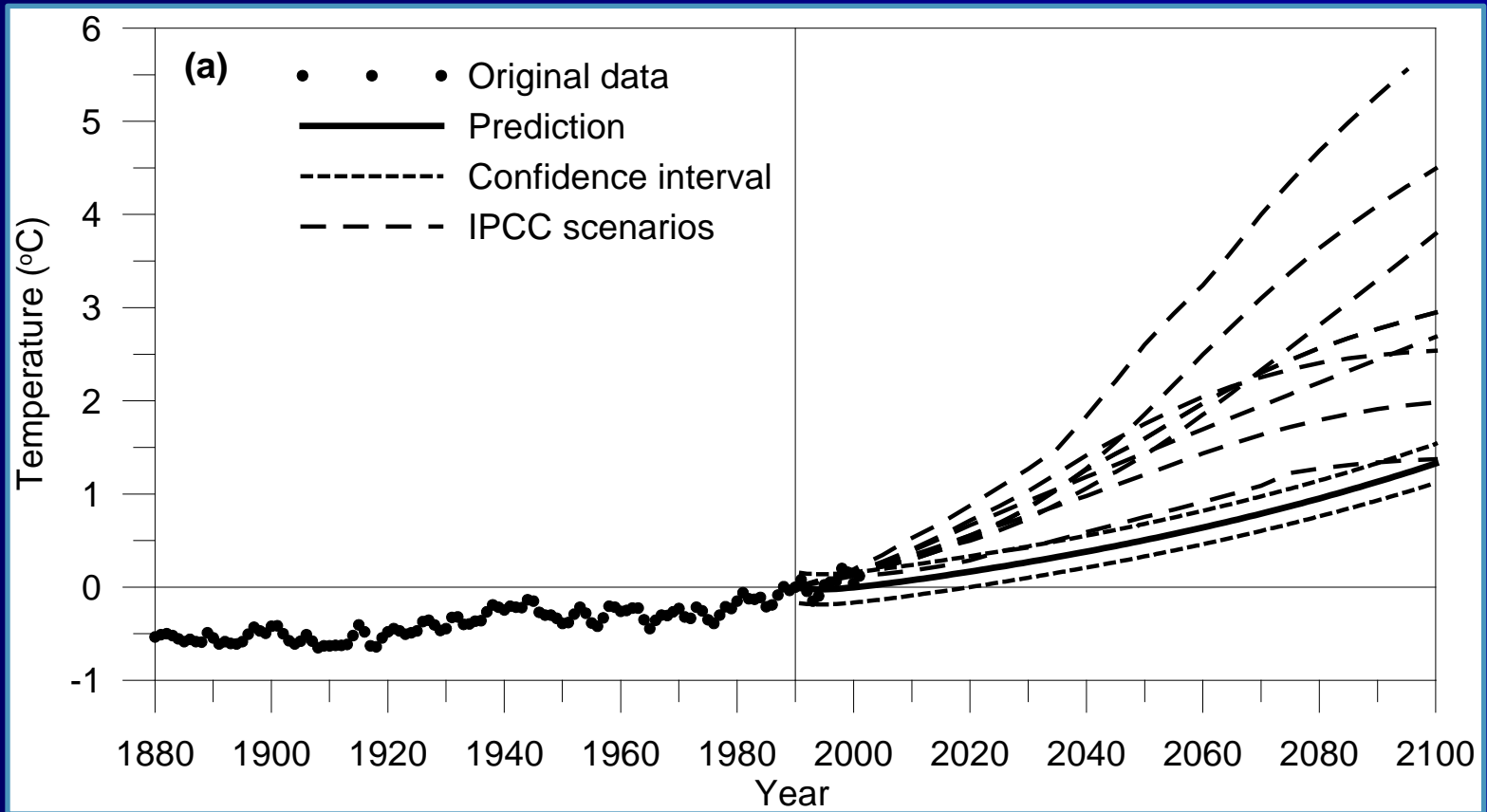

$R^2 = 0.5$

Sea-Level (1950-2001): (90%) Conf. Int.

$R^2 = 0.9$
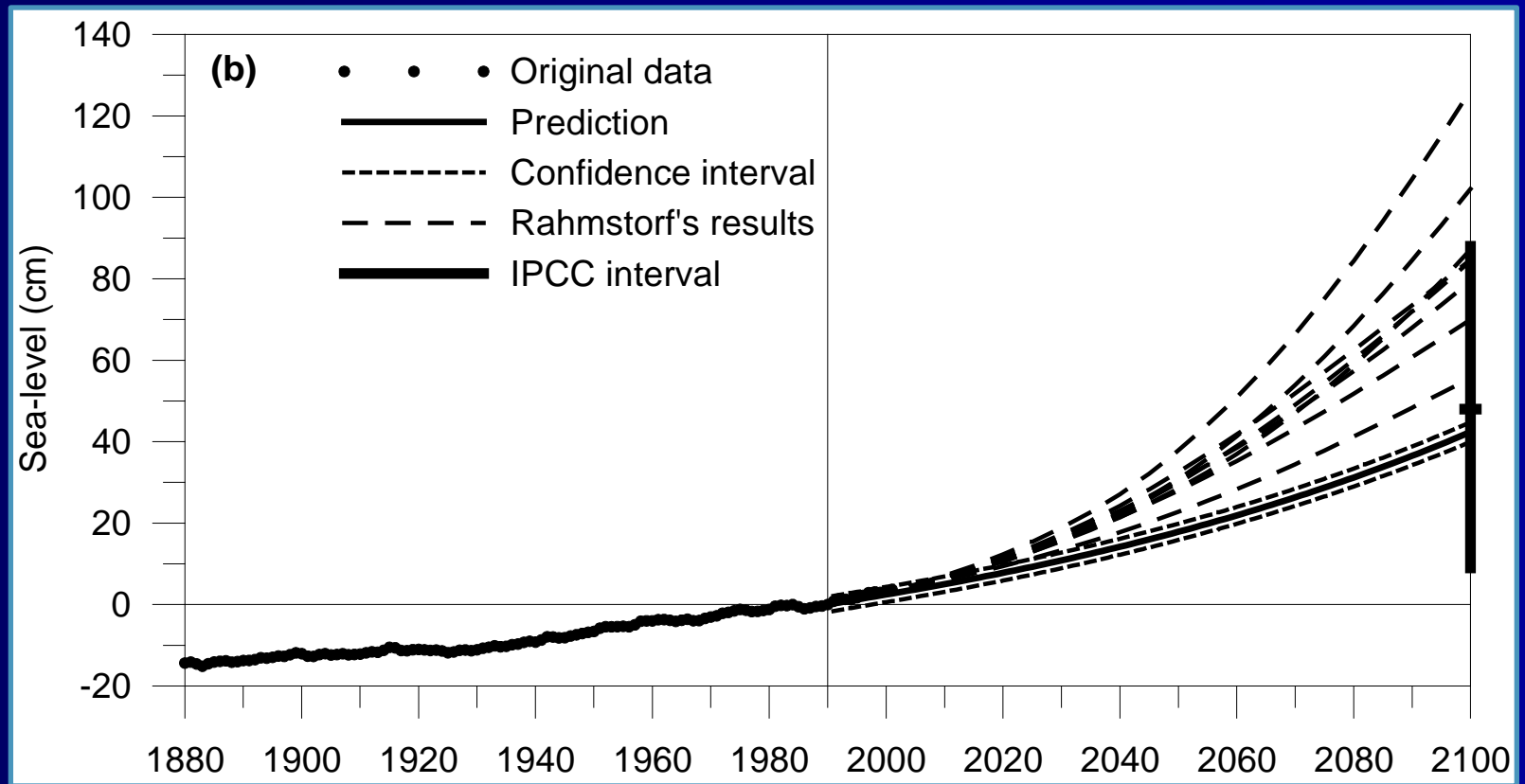
Temperature (1990-2100)

with IPCC scenarios:

Sea-Level (1990-2100)
with IPCC scenarios:

Sea-Level (1990-2100)
with IPCC scenarios: